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GEPNER TYPE STABILITY CONDITION AND KUZNETSOV EQUIVALENCE

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Abstract. This is a proceeding article of my talk 'Gepner type stability condition and Kuznetsov equivalence' at the Kinosaki Algebraic Geometry Symposium 2013.

1. Introduction

1.1. **Conjectural Gepner type stability conditions.** Historically, it has been observed that there is a curious relationship among two kinds of algebraic varieties: cubic fourfolds and K3 surfaces (cf. [BD85], [Voi86], [Has00], [Kuz10]). Our purpose is to apply the above classical observation to a modern problem in graded matrix factorizations.

Definition 1.1. *For a homogeneous polynomial*

$$W \in A := \mathbb{C}[x_1, x_2, \cdots, x_n]$$

of degree d, a graded matrix factorization consists of data

$$P^0 \xrightarrow{p^0} P^1 \xrightarrow{p^1} P^0(d)$$

where P^i are graded free A-modules of finite rank, p^i are homomorphisms of graded A-modules, $P^i \mapsto P^i(1)$ is the shift of the grading, satisfying $p^1 \circ p^0 = p^0 \circ p^1 = \cdot W$.

The homotopy category $\mathrm{HMF}(W)$ of graded matrix factorizations of W has a structure of a triangulated category. In general, there is the notion of stability conditions on triangulated categories by Bridgeland:

Definition 1.2. ([Bri07]) *A stability condition σ on a triangulated category $\mathcal{D}$ consists of a pair $(Z, \{\mathcal{P}(\phi)\}_{\phi \in \mathbb{R}})$*

$$Z \colon K(\mathcal{D}) \to \mathbb{C}, \quad \mathcal{P}(\phi) \subset \mathcal{D}$$

where Z is a group homomorphism (called central charge) and $\mathcal{P}(\phi)$ is a full subcategory (called σ-semistable objects with phase ϕ) satisfying the following conditions:

- *For $0 \neq E \in \mathcal{P}(\phi)$, we have $Z(E) \in \mathbb{R}_{>0} \exp(\sqrt{-1}\pi\phi)$.*
- *For all $\phi \in \mathbb{R}$, we have $\mathcal{P}(\phi + 1) = \mathcal{P}(\phi)[1]$.*
- *For $\phi_1 > \phi_2$ and $E_i \in \mathcal{P}(\phi_i)$, we have $\mathrm{Hom}(E_1, E_2) = 0$.*

- *For each $0 \neq E \in \mathcal{D}$, there is a collection of distinguished triangles*

$$E_{i-1} \to E_i \to F_i \to E_{i-1}[1], \quad E_N = E, \ E_0 = 0$$

with $F_i \in \mathcal{P}(\phi_i)$ and $\phi_1 > \phi_2 > \cdots > \phi_N$.

Let τ be the autequivalence of $\mathrm{HMF}(W)$ sending $P^\bullet$ to $P^\bullet(1)$. We are interested in constructing a specific type of a Bridgeland stability condition on $\mathrm{HMF}(W)$, which has a symmetric property with respect to τ. It is formulated in the following conjecture [Wal05], [KST07], [Todc]:

Conjecture 1.3. *There is a Bridgeland stability condition*

$$\sigma_G = (Z_G, \{\mathcal{P}_G(\phi)\}_{\phi \in \mathbb{R}})$$

on $\mathrm{HMF}(W)$, *where the central charge Z_G is given by*

$$Z_G\left(\bigoplus_{i=1}^{N} A(m_i) \rightleftarrows \bigoplus_{i=1}^{N} A(n_i)\right) = \sum_{i=1}^{N}\left(e^{\frac{2\pi m_i\sqrt{-1}}{d}} - e^{\frac{2\pi n_i\sqrt{-1}}{d}}\right).$$

and the set of semistable objects satisfy $\tau\mathcal{P}_G(\phi) = \mathcal{P}_G(\phi + 2/d)$.

An expected stability condition in the above conjecture was called *Gepner type* in [Todb], as we will explain below.

1.2. **Motivation for Conjecture 1.3.** Below we explain the motivation of the above conjecture. Given a triangulated category $\mathcal{D}$, the set of Bridgeland stability conditions on $\mathcal{D}$ is known to form a complex manifold. If $\mathcal{D} = D^b\,\mathrm{Coh}(X)$ for a Calabi-Yau manifold X, then the space of stability conditions $\mathrm{Stab}(X)$ is expected to be related to the stringy Kähler moduli space $\mathcal{M}_K$ of X, that is the moduli space of complex structures of a manifold $X^\vee$ mirror to X. More precisely, the space $\mathcal{M}_K$ is conjectured to be embedded into the double quotient stack

$$[\mathrm{Auteq}(X) \backslash \mathrm{Stab}(X)/\mathbb{C}]$$

via solutions of Picard-Fuchs equations which the period integrals on $X^\vee$ satisfy. For instance if $X = (W = 0)$ is a quintic 3-fold in $\mathbb{P}^4$, then $\mathcal{M}_K$ is given by the quotient stack

$$\mathcal{M}_K = \left[\{\psi \in \mathbb{C} : \psi^5 \neq 1\}/\mu_5\right].$$

The point $\psi = \infty$ is called *large volume limit*, the point $\psi^5 = 1$ is called *conifold point* and the point $\psi = 0$ is called *Gepner point*, as described in Figure 1. We refer to [Toda] for the conjectural description of the embedding map.

Near the large volume limit, Bridgeland stability conditions are expected to be approximations of the classical Gieseker stability condition on $\mathrm{Coh}(X)$. On the other hand, the stability condition at the Gepner

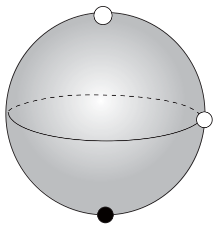

FIGURE 1. Stringy Kähler moduli space of a quintic 3-fold

point is expected to be a stability condition in Conjecture 1.3 (which is presumably unique in some sense) via Orlov equivalence [Orl09]

$$\mathrm{HMF}(W) \xrightarrow{\sim} D^b \operatorname{Coh}(X).$$

The name 'Gepner type' in [Todc] comes from the above expectation.

Another motivation comes from Donaldson-Thomas (DT) theory in [Tho00]. If there exists a desired stability condition σ_G in Conjecture 1.3, where W is a defining equation of a quintic 3-fold, then it should define the DT type invariant

$$\mathrm{DT}_G(\gamma) \in \mathbb{Q}, \quad \gamma \in \mathrm{HH}_0(W)$$

which counts σ_G-semistable graded matrix factorizations $P^\bullet$ satisfying $\mathrm{ch}(P^\bullet) = \gamma$. Here $\mathrm{HH}_0(W)$ is the Hochschild homology of $\mathrm{HMF}(W)$, and $\mathrm{ch}(*)$ is the Chern character map for graded matrix factorizations [PV12]. The Gepner type property in Conjecture 1.3 yields $\mathrm{DT}_G(\gamma) = \mathrm{DT}_G(\tau_* \gamma)$ which, together with the wall-crossing arguments [JS12], [KS], imply a non-trivial constraint among classical DT invariants on a quintic 3-fold. We expect that such a constraint is useful in computing DT invariants, and proving automorphic properties of the generating series of DT invariants predicted in string theory.

1.3. **Main result.** It has turned out that proving Conjecture 1.3 is a hard problem. A crucial issue is that there is no natural heart of a t-structure on $\mathrm{HMF}(W)$ which is intrinsic with respect to graded matrix factorizations. So far Conjecture 1.3 is known in the following cases: $n = 1$ [Tak], $d < n = 3$ [KST07], $n \le d \le 4$ [Todc], and some other weighted cases [KST07], [Todc]. The case $n = d = 5$ is the quintic 3-fold case, and we are not able to prove Conjecture 1.3 in this case at this moment. The strategy in [Todc] was to apply Orlov's result [Orl09] which relates $\mathrm{HMF}(W)$ with $D^b \operatorname{Coh}(X)$ for $X = (W = 0) \subset \mathbb{P}^{n-1}$, and construct desired stability conditions in the geometric side.

Let us focus on the low degree cases of Conjecture 1.3. It is almost trivial to prove it in the $d \le 2$ cases for any n, so the $d = 3$ case is the non-trivial lowest degree case.

Theorem 1.4. ([Todb]) *Conjecture 1.3 is true in the following cases:*

- $d = 3$ *and* $n \le 5$.

- *$d = 3$, $n = 6$ and the hypersurface $(W = 0) \subset \mathbb{P}^5$ is a general cubic fourfold containing a plane.*

In the next sections, we will give an outline of the proof of the above theorem for $(d, n) = (3, 6)$ case.

1.4. **Acknoledgement.** The author thanks to the organizers of Kinosaki Algebraic Geometry Symposium 2013.

2. Orlov/Kuznetsov equivalence

2.1. **Orlov equivalence.** Let W be a homogeneous polynomial with n variables of degree d. We recall Orlov's theorem [Orl09] which relates $\mathrm{HMF}(W)$ with the derived category of coherent sheaves on the hypersurface $X := (W = 0) \subset \mathbb{P}^{n-1}$ by semiorthogonal decompositions (SOD for short). Since we only use the case of $n > d$, we give a statement in this case.

Theorem 2.1. ([Orl09, Theorem 2.5]) *If $n > d$, then there is a fully faithful embedding for each $i \in \mathbb{Z}$*

$$\Phi_i \colon \mathrm{HMF}(W) \hookrightarrow D^b \operatorname{Coh}(X)$$

and SOD

$$D^b \operatorname{Coh}(X) = \langle \mathcal{O}_X(-i-n+d+1), \cdots, \mathcal{O}_X(-i), \Phi_i \mathrm{HMF}(W) \rangle.$$

In what follows we assume that $d = 3$, $n = 6$ so that $X = (W = 0)$ is a cubic fourfold in $\mathbb{P}^5$. Let $\mathcal{D}_X$ be the semiorthogonal summand of $D^b \operatorname{Coh}(X)$ defined by

$$D^b \operatorname{Coh}(X) = \langle \mathcal{O}_X(-3), \mathcal{O}_X(-2), \mathcal{O}_X(-1), \mathcal{D}_X \rangle. \tag{1}$$

By setting $\Phi = \Phi_1$ in the above notation, Orlov's theorem gives an equivalence

$$\Phi \colon \mathrm{HMF}(W) \xrightarrow{\sim} \mathcal{D}_X. \tag{2}$$

2.2. **Geometry of cubic fourfolds containing a plane.** Let $X = (W = 0) \subset \mathbb{P}^5$ be a cubic fourfold which contains a plane P. Let

$$\sigma \colon \widetilde{X} \to X, \quad \pi \colon \widetilde{X} \to \mathbb{P}^2$$

be the blow-up at P, the linear projection from P, respectively. The morphism π is a quadric fibration in the projectivization of the rank four vector bundle on $\mathbb{P}^2$, given by

$$E = \mathcal{O}_{\mathbb{P}^2}^{\oplus 3} \oplus \mathcal{O}_{\mathbb{P}^2}(-1).$$

The degeneration locus of π is a sextic $C \subset \mathbb{P}^2$. Let

$$f \colon S \to \mathbb{P}^2 \tag{3}$$

be the double cover branched along C. The curve C is non-singular for a general cubic fourfold containing a plane. In this case, the associated double cover S is a smooth projective K3 surface. In what follows, we assume that the cubic fourfold X is general so that C is non-singular. We denote by H a hyperplane in $\mathbb{P}^5$ pulled back to $\widetilde{X}$, and h is a hyperplane in $\mathbb{P}^2$ pulled back to $\widetilde{X}$ or S. The relevant diagram in this subsection is summarized below:

$$
\begin{tikzcd}
 & & \text{Blow-up at } P \arrow[d, dotted] & & \\
 & & \widetilde{X} \arrow[dll, "\sigma"'] \arrow[dr, "\pi"] & & \\
\mathbb{P}^5 \supset X & & & \mathbb{P}^2 & S \arrow[l, "f"'] \\
\text{Cubic fourfold} \arrow[u, dotted] & & & & \text{K3 surface} \arrow[u, dotted]
\end{tikzcd}
$$

2.3. **Sheaves of Clifford algebras and twisted K3 surfaces.** Similarly to the classical construction of Clifford algebras, the morphism π defines the sheaf of Clifford algebras on $\mathbb{P}^2$. It has an even part $\mathcal{B}_0$ and an odd part $\mathcal{B}_1$, which are described as

$$
\begin{aligned}
\mathcal{B}_0 &= \mathcal{O}_{\mathbb{P}^2} \oplus (\wedge^2 E \otimes \mathcal{O}_{\mathbb{P}^2}(-1)) \oplus (\wedge^4 E \otimes \mathcal{O}_{\mathbb{P}^2}(-2)) \\
\mathcal{B}_1 &= E \oplus (\wedge^3 E \otimes \mathcal{O}_{\mathbb{P}^2}(-1)).
\end{aligned}
$$

We also define $\mathcal{B}_i$ for $i \in \mathbb{Z}$ by the rule $\mathcal{B}_{i+2} = \mathcal{B}_i(1)$. By [Kuz08, Corollary 3.9], every sheaves $\mathcal{B}_i$ are flat over $\mathcal{B}_0$ and we have

$$\mathcal{B}_i \otimes_{\mathcal{B}_0} \mathcal{B}_j \cong \mathcal{B}_{i+j}, \quad \text{for all } i, j \in \mathbb{Z}.$$

In particular, for every i there is an equivalence of abelian categories

$$\otimes_{\mathcal{B}_0} \mathcal{B}_i \colon \operatorname{Coh}(\mathcal{B}_0) \xrightarrow{\sim} \operatorname{Coh}(\mathcal{B}_0).$$

Here $\operatorname{Coh}(\mathcal{B}_0)$ is the abelian category of coherent right $\mathcal{B}_0$-modules on $\mathbb{P}^2$.

Let S be the K3 surface obtained as a double cover (3). By [Kuz08, Section 3.5], there exists a sheaf of Azumaya algebras $\mathcal{B}_S$ on S such that $f_* \mathcal{B}_S = \mathcal{B}_0$, and an equivalence

$$f_* \colon \operatorname{Coh}(\mathcal{B}_S) \xrightarrow{\sim} \operatorname{Coh}(\mathcal{B}_0).$$

The abelian categories $\operatorname{Coh}(\mathcal{B}_0)$, $\operatorname{Coh}(\mathcal{B}_S)$ are also described in terms of twisted sheaves. There exists an element in the Brauer group

$$\alpha \in \operatorname{Br}(S) = H^2(S, \mathcal{O}_S^*), \quad \alpha^2 = \mathrm{id}$$

and an α-twisted vector bundle $\mathcal{U}_0$ of rank two such that $\mathcal{B}_S = \mathcal{E}nd(\mathcal{U}_0)$ and the functor

$$\operatorname{Coh}(S, \alpha) \ni F \mapsto \mathcal{U}_0^\vee \otimes F \in \operatorname{Coh}(\mathcal{B}_S)$$

is an equivalence. Here $\mathrm{Coh}(S,\alpha)$ is the abelian category of α-twisted coherent sheaves on S (cf. [HS05, Section 1]). Combined with the above equivalences, we obtain the equivalence

$$\Upsilon(-) := f_*(\mathcal{U}_0^\vee \otimes -)\colon D^b\,\mathrm{Coh}(S,\alpha) \xrightarrow{\sim} D^b\,\mathrm{Coh}(\mathcal{B}_0). \tag{4}$$

2.4. **Orlov/Kuznetsov equivalence.** Let $\mathcal{D}_X$ be the semiorthogonal summand of $D^b\,\mathrm{Coh}(X)$ given by (1). In [Kuz10], Kuznetsov established an equivalence between $D^b\,\mathrm{Coh}(\mathcal{B}_0)$ and $\mathcal{D}_X$. A starting point is the fully faithful functor

$$\Psi\colon D^b\,\mathrm{Coh}(\mathcal{B}_0) \to D^b\,\mathrm{Coh}(\widetilde{X})$$

constructed in [Kuz08], defined as a Fourier-Mukai transform

$$\Psi(-) = \pi^*(-) \otimes_{\pi^*\mathcal{B}_0} \mathcal{E}.$$

Here $\mathcal{E}$ is a sheaf of left $\pi^*\mathcal{B}_0$-modules on $\widetilde{X}$ given by the cokernel of the canonical surjection

$$\pi^*\mathcal{B}_0(-2H) \to \pi^*\mathcal{B}_1(-H) \to \mathcal{E} \to 0.$$

As $\mathcal{O}_{\widetilde{X}}$-module, the sheaf $\mathcal{E}$ is locally free of rank four. Kuznetsov [Kuz10] performs a sequence of mutations of SOD of $D^b\,\mathrm{Coh}(\widetilde{X})$, and proves the following result:

Theorem 2.2. ([Kuz10]) *The functor*

$$\Theta\colon D^b\,\mathrm{Coh}(\mathcal{B}_0) \to \mathcal{D}_X$$

given by

$$\begin{aligned}\Theta(F) = \mathrm{Tot}\{\mathbf{R}\,\mathrm{Hom}(\mathcal{O}_{\widetilde{X}}(h-H), \Psi(F)) \otimes I_P \to \mathbf{R}\sigma_*\Psi(F) \\ \to \mathbf{R}\,\mathrm{Hom}(\Psi(F), \mathcal{O}_{\widetilde{X}}(-h))^\vee \otimes \mathcal{O}_X(-1)\}.\end{aligned}$$

is an equivalence. Here $I_P \subset \mathcal{O}_X$ is the ideal sheaf of P.

We summarize the equivalences obtained so far in the following corollary:

Corollary 2.3. *There is a sequence of equivalences*

$$D^b\,\mathrm{Coh}(S,\alpha) \xrightarrow{\Upsilon} D^b\,\mathrm{Coh}(\mathcal{B}_0) \xrightarrow{\Theta} \mathcal{D}_X \xleftarrow{\Phi} \mathrm{HMF}(W).$$

Here Υ is given in (4), Θ is given in Theorem 2.2 and Φ is given in (2).

3. Outline of the proof of Theorem 1.4 for $(d, n) = (3, 6)$

Step 1. *Description of the grade shift functor.*

Our first step is to describe the grade shift functor τ on $\mathrm{HMF}(W)$ in terms of $D^b \operatorname{Coh}(\mathcal{B}_0)$. We define the autequivalence F_B of $D^b \operatorname{Coh}(\mathcal{B}_0)$ to be

$$F_B := \mathrm{ST}_{\mathcal{B}_1}^{-1} \circ \otimes_{\mathcal{B}_0} \mathcal{B}_{-1}[1].$$

Here $\mathrm{ST}_{\mathcal{B}_1}$ is the Seidel-Thomas twist [ST01] associated to $\mathcal{B}_1$:

$$\mathrm{ST}_{\mathcal{B}_1}(-) = \operatorname{Cone}\left(\mathbf{R}\operatorname{Hom}(\mathcal{B}_1, -) \otimes \mathcal{B}_1 \to -\right).$$

We have the following proposition:

Proposition 3.1. ([Todb, Corollary 3.4]) *The following diagram commutes:*

$$\begin{tikzcd} D^b \operatorname{Coh}(\mathcal{B}_0) \arrow[r, "\Phi^{-1}\circ\Theta"] \arrow[d, "F_B"'] & \mathrm{HMF}(W) \arrow[d, "\tau"] \\ D^b \operatorname{Coh}(\mathcal{B}_0) \arrow[r, "\Phi^{-1}\circ\Theta"] & \mathrm{HMF}(W). \end{tikzcd}$$

The above proposition is proved in the following way: the functor τ is described in terms of $\mathcal{D}_X$ under Orlov equivalence Φ as $F_X = \mathrm{ST}_{\mathcal{O}_X} \circ \otimes \mathcal{O}_X(1)$ by [BFK12]. It is enough to prove the commutativity of

$$\begin{tikzcd} D^b \operatorname{Coh}(\mathcal{B}_0) \arrow[r, "\Theta"] \arrow[d, "F_B"'] & \mathcal{D}_X \arrow[d, "F_X"] \\ D^b \operatorname{Coh}(\mathcal{B}_0) \arrow[r, "\Theta"] & \mathcal{D}_X. \end{tikzcd}$$

The above commutativity is proved by proving the commutativity for objects of the form $\Upsilon(\mathcal{O}_x)$ with $x \in S$, and the commutativity of some numerical classes of objects.

Step 2. *Description of the central charge Z_G.*

The next step is to describe the central charge Z_G in terms of α-twisted sheaves on the K3 surface S. Recall that by Corollary 2.3, there is a sequence of equivalences

$$D^b \operatorname{Coh}(S, \alpha) \xrightarrow{\Upsilon} D^b \operatorname{Coh}(\mathcal{B}_0) \xrightarrow{\Theta} \mathcal{D}_X \xleftarrow{\Phi} \mathrm{HMF}(W).$$

Let Z_G be the canonical central charge on $\mathrm{HMF}(W)$ given in Conjecture 1.3. We compute the pull-back of the central charge Z_G on $\mathrm{HMF}(W)$ by the above sequence of equivalences, using the result of Proposition 3.1. The resulting central charge on $D^b \operatorname{Coh}(S, \alpha)$ coincides with an integral over S which appeared in Bridgeland's paper [Bri08]:

Proposition 3.2. ([Todb, Proposition 4.7]) *There is an element $B \in H^{1,1}(S, \mathbb{Q})$ and $c \in \mathbb{C}^*$ such that we have*

$$Z_G \circ \Phi^{-1} \circ \Theta \circ \Upsilon(E) = c \cdot \int_S e^{B - \frac{\sqrt{-3}}{4} h} \operatorname{ch}(E) \sqrt{\operatorname{td}_S}$$

for any $E \in D^b \operatorname{Coh}(S, \alpha)$. Here h is a hyperplane in $\mathbb{P}^2$ pulled back to S.

The Chern character on $D^b(S, \alpha)$ is the *untwisted* Chern character, defined to be the twisted Chern character by Huybrechts-Stellari [HS05], multiplied by the exponential of the minus of the B-field to get back to the untwisted one. Although it takes its value in an algebraic class, it is no longer defined in the integer coefficient.

Step 3. *Construction of a Gepner type stability condition.*

The final step is to construct a corresponding Gepner type stability condition on $D^b \operatorname{Coh}(S, \alpha)$, using the above descriptions of the grade shift functor and the central charge. In this step, we need a further genericity assumption: the Brauer class α is non-trivial. This condition is not satisfied only if X lies in a union of countable many hypersurfaces in the moduli space of cubic fourfolds containing a plane. Let Z'_G be the central charge on $D^b \operatorname{Coh}(S, \alpha)$ defined by

$$Z'_G(E) = -\int_S e^{B - \frac{\sqrt{-3}}{4} h} \operatorname{ch}(E) \sqrt{\operatorname{td}_S}.$$

By the arguments so far, the following result obviously implies Theorem 1.4 as desired:

Theorem 3.3. ([Todb, Theorem 4.13]) *Suppose that $\alpha \neq 1$. Then there is a Bridgeland stability condition $\sigma'_G = (Z'_G, \{\mathcal{P}'(\phi)\}_{\phi \in \mathbb{R}})$ on $D^b \operatorname{Coh}(S, \alpha)$ satisfying*

$$\Upsilon^{-1} \circ F_B \circ \Upsilon \circ \mathcal{P}'(\phi) = \mathcal{P}'(\phi + 2/3).$$

The proof relies on a standard technique on constructions of stability conditions on K3 surfaces [Bri08]. It requires proving the non-existence of 'bad' spherical objects, in which we use the $\alpha \neq 1$ assumption.

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